Matrix Equations and Inverses

Finite Math

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Quiz

Is this a square matrix?

$$\begin{bmatrix} 1 & 2 & 4 \\ 8 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Matrix Equation

Example

Find a, b, c, and d such that

$$\begin{bmatrix} 6 & -5 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -16 & 64 \\ 24 & -6 \end{bmatrix}$$

Now You Try It!

Example

Find a, b, c, and d such that

$$\begin{bmatrix} 6 & -5 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The Identity Matrix

Example

Find the products

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ *and* $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ *and* $\begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The Identity Matrix

Definition (Identity Matrix)

The $n \times n$ identity matrix is a matrix, denoted by I or I_n, which is an $n \times n$ matrix with 1's on the primcipal diagonal and 0's everywhere else. For an $m \times n$ matrix M, we have

$$I_m M = M = M I_n.$$

From the above example, we have the two forms of the identity matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse of a Square Matrix

It is only possible to find a multiplicative inverse of a matrix if it is a square matrix. So, we now restrict ourselves to square matrices.

Definition (Inverse Matrix)

If M is a square matrix of size n, and if there is a matrix, denoted M^{-1}, such that

$$MM^{-1} = M^{-1}M = I_n$$

we call M^{-1} the inverse of M. If M does not have an inverse, then M is called a singular matrix.

Example

Find the inverse of the matrix

$$M = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

Inverse of a Square Matrix

Example

Find the inverse of the matrix

$$N = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

Inverse of a Square Matrix

Example

Find the inverse of the matrix

$$M = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 2 & -3 \\ -2 & -3 & -1 \end{bmatrix}.$$

Inverse of a Square Matrix

Theorem

To find the inverse of an $n \times n$ matrix M, one begins with the augmented matrix $[M|\, I_n]$ and uses row operations to transform it into $\left[\, I_n |\, M^{-1} \right]$. However, if one or more rows of all 0's appear on the left side of the augmented matrix, M is not invertible, i.e., M^{-1} does not exist.

Now You Try It!

Example

Find the inverse of the following matrices (if possible):

(a)

$$M = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

(b)

$$P = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

(c)

$$N = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

A Useful Trick

Remark

There is a trick to invert a 2×2 *matrix. If* $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *then*

$$M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$